ON THE VELOCITY FIELD FORMED BY A FLAT DIE PRESSING ON A PLASTIC HALF-SPACE

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PMM Vol. 25, No. 3, 1961, pp. 552-553

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(Received December 26, 1960)

The problem of a flat die pressing on a half-space was first considered by Prandtl [1]; Hill [2] found another solution of this problem; later Prager [3] indicated the possibilities of constructing solutions which are combinations of the Hill and Prandtl solutions. Below are considered additional possibilities of constructing the velocity field.

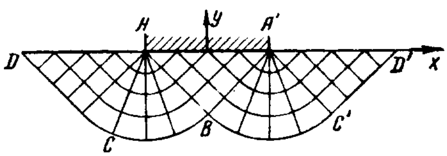

Fig. 1.

In the triangle ABA' (Fig. 1) the slip lines are straight lines, and the Geiringer relationships [4] have the following form:

$$\frac{\partial v_\alpha}{\partial \alpha} = 0, \qquad \frac{\partial v_\beta}{\partial \beta} = 0$$

hence

$$v_\alpha = f(\beta), \qquad v_\beta = \varphi(\alpha)$$

where v_α are the velocity components along the curves $\beta =$ const, and v_β along $\alpha =$ const.

In the region $ABCD$, $v_\beta = 0$ and $v_\alpha =$ const along the curves $\alpha =$ const; moreover, the Geiringer equations are satisfied identically. Solutions in the region $A'B'C'D'$ are constructed in an analogous way.

Let us specify one of the velocity components along AA', for example v_0. Assuming that the die moves downward as a rigid body, we have

$$v_y = -1 \quad \text{on } AA' \tag{1.1}$$

From (1.1) along AA' the second velocity component $v_\beta = \sqrt{2} \times (1 - v_\alpha)$ is determined. Thus, the values of the velocity components v_α and v_β in the plastic region are completely determined.

Hill's solution (Fig. 2) corresponds to the following stress distribution along AA':

$$v_\alpha = \sqrt{2}, \qquad v_\beta = 0 \quad \text{for } x < 0$$

$$v_\alpha = 0, \qquad v_\beta = \sqrt{2} \quad \text{for } x > 0$$

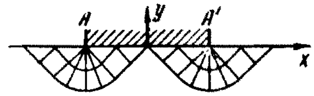

Fig. 2.

At $x = 0$ the velocities remain undetermined. In the Prandtl solution we have along AA'.

$$v_\alpha = \frac{\sqrt{2}}{2}, \qquad v_\beta = \frac{\sqrt{2}}{2}$$

All previous solutions differ from one another in the assumption of the location of the rigid-plastic boundary. Except for the Hill's solution, the velocity field is not determined by the assumed location of a rigid-plastic boundary.

The existence of a possibility of such a solution will be demonstrated for a combined solution obtained by the following velocity distribution on the boundary AA' (Fig. 3):

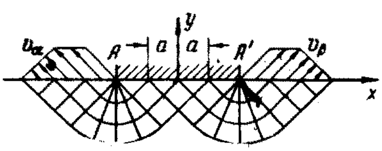

Fig. 3.

$$v_\alpha = \frac{\sqrt{2}}{2}, \quad v_\beta = \frac{\sqrt{2}}{2} \quad \text{for } -a \leqslant x \leqslant a$$

$$v_\alpha = \sqrt{2}, \quad v_\beta = 0 \quad \text{for } x < -a$$

$$v_\alpha = 0, \quad v_\beta = \sqrt{2} \quad \text{for } x > a$$

It is possible to construct other solutions. A possible velocity distribution in a plastic region is obtained, for instance, setting on AA'

$$v_\alpha = \frac{\sqrt{2}\,(a - x)}{2a}, \quad v_\beta = \frac{\sqrt{2}\,(a + x)}{2a} \quad \text{for } -a \leqslant x \leqslant a$$

$$v_\alpha = \sqrt{2}, \quad v_\beta = 0 \quad \text{for } x < -a, \qquad v_\alpha = 0, \quad v_\beta = \sqrt{2} \quad \text{for } x > a$$

In the Hill solution the region of the multivaluedness of the velocity field is reduced to a point $x = 0$ (see Fig. 2).

We note that for an infinitely small curvature of the free boundary the construction of the Prandtl solution is impossible. Indeed, the perturbations of the slip-lines net in ABA' will be determined on one side by the perturbations of the segments AD, and on the other side by the perturbations of the segments $A'D'$. Generally speaking, the perturbations of the characteristic lines in ABA' will be different, and the construction of a unique net is impossible. Hill's solutions can be constructed for arbitrary perturbations of the boundary.

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Translated by R.M. E.-I.